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The market penetration and compliance rate of Advanced Traveler Information Systems under dynamic traffic conditions

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Abstract: We consider the objective of ATIS is to reduce travelers' travel cost uncertainty with recurrent network congestion through provision of traffic information. Most of studies focus on the use of static model for an essentially dynamic problem. We propose a multi-class stochastic dynamic path and departure time model for modeling interactions among travelers in general networks with ATIS. A nested-logit model is used for the determination of the market penetration and compliance rate of ATIS. The upper level is for market penetration; and the lower level for compliance rate. Then an iterative algorithm is presented for the determination of endogenous market penetration and compliance rate of ATIS and the equilibrium network flow patterns. Finally, the model and algorithm are tested in a simple network.

Key Words: Advanced Traveler Information Systems; Market Penetration; Compliance Rate; Nested Logit Model

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1. Introduction

Advanced traveler information systems, as a major part of Intelligent Transportation Systems, are generally believed to be efficient means for improving individual traveler's trip planning, alleviating traffic congestion and enhancing traffic network performance. There are many researchers in modeling and evaluating the impacts of ATIS on travelers and transportation systems in order to determine the feasibility, risks and benefits of such technology. Up to now, These studies include path choice surveys (Abdel.Aty,M.A etc,1997, E.Hato,etc, 1999),field deployments(Tsuji etc,1985),laboratory experiments(Yang 1993, Reddy 1995),computer simulations(Emmerink 1995, Mahmassani 1994,1999) and analytical models (Yang Hai 1998,1999,YaFeng Yin 2003, H.Lo 2002a,William 2003).

The path choice of travelers equipped and unequipped with ATIS is actually the most important aspect of ATIS. The difference of the information received by travelers will lead to different travel choice behaviors. According to previous studies, it is considered that equipped travelers will follow SO, deterministic user equilibrium or Stochastic user equilibrium (smaller perception uncertainty) and unequipped travelers will follow SUE(higher perception uncertainty) (Kanafani and Al-Deek, 1991; Van Vuren & Walting 1991; Benneet, 1993; Yang, 1998; Lo et al., 1999; Yang and Meng, 2001; Yin and Yang, 2003).

The market penetration of ATIS, defined as the proportion of vehicles (travelers) equipped with ATIS, has widely been recognized as an important factor to determine the actual advantage of ATIS implementation. The exogenous market penetration was used for evaluating the benefits of ATIS in some previous studies (Yang 1999, Emmerink 1995), regardless of the individual's trip cost or potential travel cost saving. Yang (1998) was first to propose the endogenous market penetration model in which the market penetration is dependent upon the information benefit from ATIS. Yang (2001) further modeled the time line of the growth of market penetration to reach final stationary equilibrium based on a modified logistic type growth model. Similar studies were carried out by Lo and Szeto(1999,2002a), Yin(2003).

The compliance rate of ATIS, defined as the proportion of complied travelers in total equipped travelers, is another important factor that influences the benefits of ATIS, but it has received little attention. Previous researches have assumed perfect compliance of travelers with ATIS. However, although ATIS are intended to provide accurate real-time traffic information to travelers, it is doubtful that whether travelers would all comply with these systems. There are many factors that influence equipped travelers' compliance such as traveler individual characteristics, reliability of ATIS and so on. Boehm Davis and Fox 1998, Chen etc 1999,Oh etc 2001 have carried out the researches for compliance rate of ATIS, respectively. Yin (2003) considered simultaneous determination of equilibrium market penetration and compliance rate of ATIS using a multiple behavior equilibrium model.

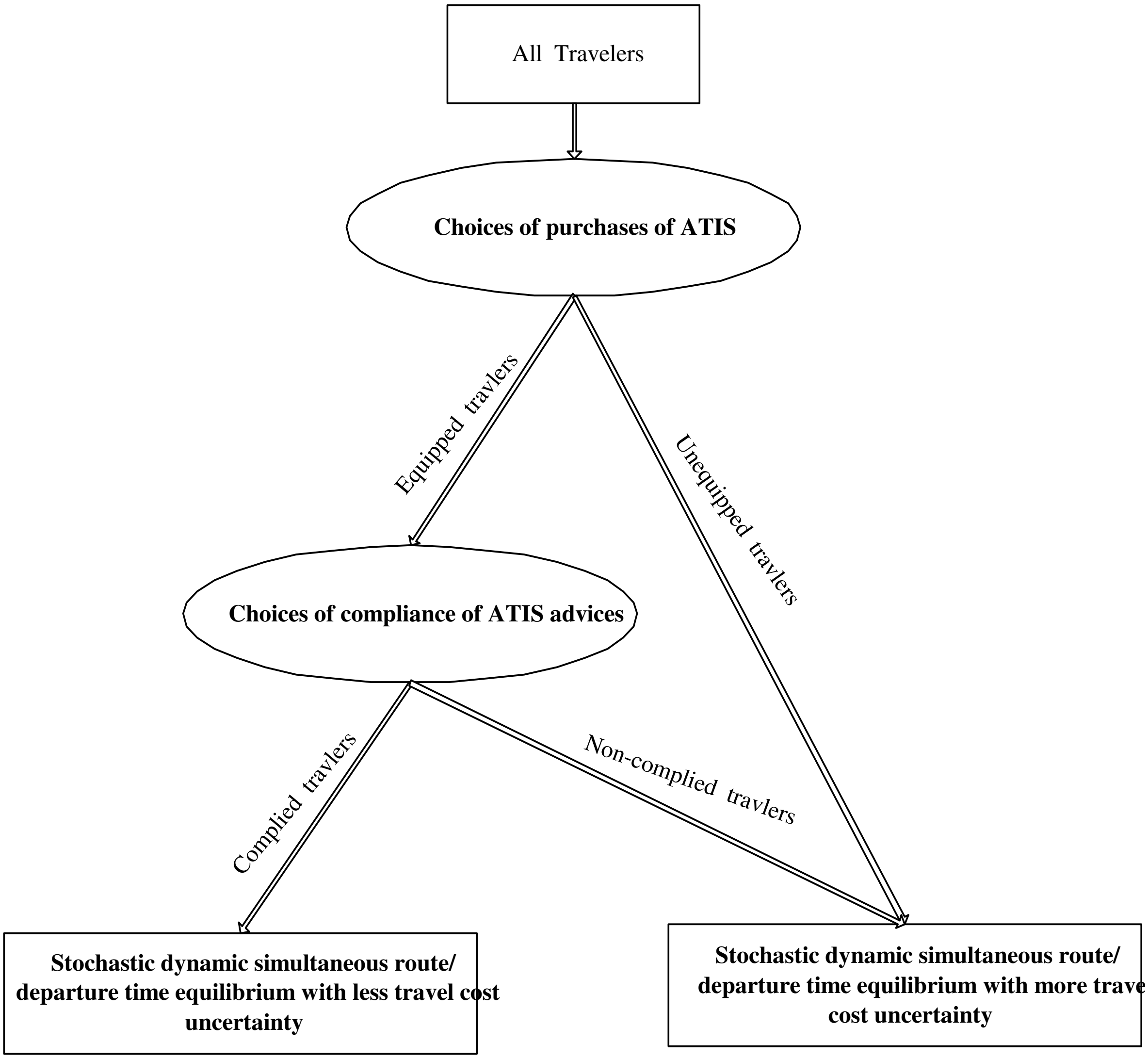

Fig. 1. Multiple user behaviors in an ATIS environment.

However, most of the previous models for assessing the impacts of ATIS are static. Generally, these models don't permit the study of other ATIS impacts such as changes in departure time, dynamic queuing and so on, which may result in unconformity with actual situations. Emmerink (1995) has contributed substantially in studying the economic impacts of driver information system by using microscopic simulations. Mahmassani etc(1994) used DYNASMART model to assess the effects of ATIS. Lo (2002b) developed a dynamic traffic assignment model, developed a dynamic traffic assignment model, and made a contrast of the impacts upon ATIS benefits between the static model and the dynamic model. Huang and Wiliam (2004) proposed a multi-class dynamic user equilibrium model with ATIS, in which such

problems as departure time and network queue conditions are taken into consideration. Similar studies were carried out by B.Ran(1996), Hong K.Lo(1996). However they didn't consider the endogenous market penetration and compliance rate of ATIS.
In this paper, we extend Ying (2003)'s static ATIS model to a dynamic one by utilizing the dynamic traffic network model developed by Chabini (2001). We classify travelers into two classes; Class (i) represents unequipped travelers and equipped travelers who do not follow the ATIS advice and hence have partial traffic information (perhaps from past experiences). It is assumed that this class of travelers would choose their paths and departure times based on *perceived* rather than actual travel costs. Class (ii) represents equipped travelers who comply with ATIS advice and therefore can receive real-time traffic information but that is also partial or imperfect. Further, this class of travelers is assumed to determine their travel choices in a stochastic manner also, but with lower travel cost variation than Class (i). Thus, two specific classes of travelers are considered in the dynamic network equilibrium model, as illustrated in Fig. 1. In fact, travel cost is the most important factor influencing the travelers' path and departure time choices, thus the rational assumption of actual travel cost saving will ascertain whether a traveler purchases an ATIS device or not. If equipped, it should be determined whether a traveler follows the advice or not. Here, A nested-logit model is used in determining the equilibrium market penetration and compliance rate of ATIS, in which the upper level depicts travelers' purchase behaviors, and the lower level describes compliance behaviors of equipped travelers. With these considerations, we propose an iterative procedure to calculate the endogenous equilibrated market penetration and compliance rate of ATIS.

The other parts of this paper are organized as follows. In section 2, we propose a multi-class dynamic simultaneous path/departure time equilibrium model to describe path and departure time choices of complied and unequipped & non-complied travelers. In section 3 the endogenous market penetration and compliance rate of ATIS are determined simultaneously by using the nested-logit model. In section 4, an iterative algorithm is developed and illustrated with a numerical example in section 5.

2. Multi-class stochastic dynamic path and departure time equilibrium

Consider a network G =($N; A$), where N is the set of nodes and A is the set of links in the network. Let a denote a link of the network connecting a pair of nodes (i,j) and let p denote a path that is consisted of a series of directed link ($a_1,a_2,\ldots,a_n$) between origin r and destination s. Let RS denote the set of all OD pairs in the network. P_{rs} denotes the set of paths between OD pair $rs\in$ RS and the entire set of paths in the network by P. Time horizon T is divided by a finite number of time interval $k\in\{1,2,\cdots,n\}$. Let δ be the length of time interval. In other words, we have n time intervals; index k represents the time interval $[(k-1)\cdot\delta,k\cdot\delta)$. Here, we assume the study horizon is long enough to ensure all travelers can exit from the network after the time T. on the other hand, it is also assumed that the value of δ is small enough so that the discrete-time model can approximate its continuous time counterpart.

Here, we assume complied travelers will follow stochastic dynamic simultaneous path/departure time equilibrium (SUE-SRD) with smaller travel cost uncertainty. However, unequipped and equipped but un-complied travelers follow stochastic dynamic equilibrium with higher travel cost uncertainty, because they choose (called time-dependent paths later) path and departure time according to their trip habits or past experiences. In this paper, the SUE-SRD simultaneous path and departure time choice problem is modeled by a nested logit formulation and further described by an equivalent VI formulation.

$\hat{f}_p^{rs}(k)$, $\tilde{f}_p^{rs}(k)$ the inflow rate of complied and unequipped & non-complied travelers entering path *p* between OD pair *rs* during interval *k*

$\hat{f}$, $\tilde{f}$ the set of $\{\hat{f}_m^{rs}(k), \forall rs, p, k\}$ $\{\tilde{f}_p^{rs}(k), \forall rs, p, k\}$

$\hat{q}^{rs}$, $\tilde{q}^{rs}$ the demand of complied and unequipped & non-complied travelers between OD pair *rs*

$\hat{C}_p^{rs}(k,\cdot)$, $\tilde{C}_p^{rs}(k,\cdot)$ the perceived unit travel cost of complied and unequipped & non-complied travelers between OD pair *rs* selecting path *p* and departure time *k.*

$\hat{\theta}$, $\tilde{\theta}$ the parameter representing travel cost variation of complied, unequipped & non-complied travelers

$c_p^{rs}(k,\cdot)$ the unit travel cost (on an average) incurred by travelers departing from *r* and selecting path *p* to *s* during interval *k*.

Complied travelers follow the stochastic dynamic simultaneous path and departure time equilibrium (SUE-SRD), expressed as

$$\hat{f}_p^{rs}(k) = \frac{\exp(-\hat{\theta} \cdot c_p^{rs}(k,\cdot))}{\sum_p \sum_k \exp(-\hat{\theta} \cdot c_p^{rs}(k,\cdot))} \cdot \hat{q}^{rs} \quad \forall rs, p, k \tag{1}$$

Where $\hat{\theta}$ is the parameter representing general travel cost perception variation of the complied travelers. A higher $\hat{\theta}$ means smaller travel cost variation and better information quality.

The logit-based SUE-SRD of complied travelers can be expressed as follows.

$$\hat{C}_p^{rs}(k,\cdot)\begin{cases} = \hat{c}_{\min}^{rs}(\cdot) \text{ if } \hat{\mathrm{f}}_{\mathrm{p}}^{\mathrm{rs*}}(\mathrm{k})>0 \\ > \hat{c}_{\min}^{rs}(\cdot) \text{ if } \hat{\mathrm{f}}_{\mathrm{p}}^{\mathrm{rs*}}(\mathrm{k})=0 \end{cases} \quad \forall \mathrm{rs,k,p} \tag{2}$$

$$\sum_p \sum_k \hat{f}_p^{rs*}(k) = \frac{\hat{q}^{rs}}{\boldsymbol{\delta}} \qquad \forall rs \tag{3}$$

$$\hat{C}_p^{rs}(k,\cdot) = c_p^{rs}(k,\cdot) + \frac{1}{\hat{\boldsymbol{\theta}}} \ln \hat{f}_p^{rs*}(k) \;\; \forall rs k,p \tag{4}$$

$$\hat{f}_p^{rs*}(k) \geq 0 \qquad \forall rs, k, p \tag{5}$$

Where $\hat{c}^{rs}_{\min}(\cdot)$ is the minimum perceived unit travel cost of complied travelers between origin *r* and destination *s*, $\hat{c}^{rs}_{\min}(\cdot)=\min\{\hat{C}^{rs}_{p}(k,\cdot),\forall p,k\}$. $\hat{C}^{rs}_{p}(k,\cdot)$ is the perceived unit travel cost incurred by complied travelers entering path *p* between origin *r* and destination *s* during interval *k*. Equation(4) represents the flow conservation of complied travelers between origin *r* and destination *s* and equation(5) represents the non-negativity of all path inflow rates.

For complied travelers and for each origin-destination (OD) pair, the perceived path travel costs experienced for complied travelers, regarding of departure times, is equal and minimum, and less than (or equal to) the perceived path travel costs for complied travelers on any unused paths.

The above SUE-SRD equilibrium condition of complied travelers can be expressed by a finite dimensional variational inequality formulation.

Find a vector $\hat{f}^{*}\in\hat{\Omega}$ if and only if it satisfy

$$\sum_{rs}\sum_{p}\sum_{k}\hat{C}^{rs}_{p}(k,\hat{f}^{*},\tilde{f})(\hat{f}^{rs}_{p}(k)-\hat{f}^{rs*}_{p}(k))\geq 0 \quad \forall\hat{f}\in\hat{\Omega} \tag{6}$$

Where, $\hat{\Omega}$ is a closed convex.

$$\hat{\Omega}=\left\{\hat{f}\,\middle|\sum_{p}\sum_{k}\hat{f}^{rs}_{p}(k)=\frac{\hat{q}^{rs}}{\delta},\hat{f}^{rs}_{p}(k)\geq 0,\forall rs\right\}$$

The treatment of unequipped & non-complied travelers is identical. Without loss of generality, one may write:

$$\tilde{f}^{rs}_{p}(k)=\frac{\exp(-\tilde{\theta}\cdot c^{rs}_{p}(k,\cdot))}{\sum_{p}\sum_{k}\exp(-\tilde{\theta}\cdot c^{rs}_{p}(k,\cdot))}\cdot\tilde{q}^{rs} \quad \forall rs,p,k \tag{7}$$

Where $\tilde{\theta}$ expresses the travel cost perception variation of unequipped & non-complied travelers that can be interpreted as their familiarity of the network condition or the past experiences (H.Lo 2002a).

Following the above analysis; the above SUE-SRD equilibrium condition of unequipped & non-complied travelers can be expressed by a finite dimensional variational inequality formulation also.

Find a vector $\tilde{f}^{*}\in\tilde{\Omega}$ if and only if it satisfy

$$\sum_{rs}\sum_{p}\sum_{k}\tilde{C}^{rs*}_{p}(k,\hat{f},\tilde{f}^{*})(\tilde{f}^{rs}_{p}(k)-\tilde{f}^{rs*}_{p}(k))\geq 0 \quad \forall\tilde{f}\in\tilde{\Omega} \tag{8}$$

Where, $\tilde{\Omega}$ is a closed convex.

$$\tilde{\Omega}=\left\{\tilde{f}\,\middle|\sum_{p}\sum_{k}\tilde{f}^{rs}_{p}k)=\frac{\tilde{q}^{rs}}{\delta},\tilde{f}^{rs}_{p}(k)\geq 0,\forall rs\right\}$$

$\hat{q}^{rs}$ is the total demand of complied travelers for each OD pair *rs*, $\tilde{q}^{rs}$ is the total demand of unequipped & non-complied travelers for each OD pair *rs*.

$$\hat{q}^{rs} + \tilde{q}^{rs} = q^{rs} \quad \forall rs \tag{9}$$

Where q^{rs} is the total number of both complied travelers and unequipped & non-complied travelers over the network for each OD pair *rs*.

2.1 The Composite VI Formulation

The composite VI problem that integrate the VI (6) with VI (8) is equivalent to the above conditions (1) and (7)

The composite VI model can be formulated as follows:

Find a vector ($\hat{f}^* \in \hat{\Omega}, \tilde{f}^* \in \tilde{\Omega}$) that is a multi-class stochastic dynamic user equilibrium pattern if and only if it satisfies the VI problem

$$\sum_{rs}\sum_{p}\sum_{k} \tilde{C}_p^{rs}(k, \hat{f}^*, \tilde{f}^*)(\tilde{f}_p^{rs}(k) - \tilde{f}_p^{rs*}(k)) + \sum_{rs}\sum_{p}\sum_{k} \hat{C}_p^{rs*}(k, \hat{f}^*, \tilde{f}^*)(\hat{f}_p^{rs}(k) - \hat{f}_p^{rs*}(k)) \geq 0$$

$$\forall \hat{f} \in \hat{\Omega}, \forall \tilde{f} \in \tilde{\Omega} \tag{10}$$

Where $\hat{\Omega}$ and $\tilde{\Omega}$ are the sets of all feasible path inflow rates with all departure times associated with complied and unequipped & non-complied travelers, respectively.

2.2 Discrete-time dynamic network model

Here, the single class dynamic network models of Freisz(1993),chabini(2001) are extended to multi-class dynamic network model.

$\hat{u}_{ap}^{rs}(k), \tilde{u}_{ap}^{rs}(k)$ =the inflow rate of complied and unequipped & non-complied on link *a* of path *p* during interval *k,* respectively.

$\hat{v}_{ap}^{rs}(k), \tilde{v}_{ap}^{rs}(k)$ =the departure rate of complied and unequipped & non-complied travelers on link *a* of path *p* during interval *k,* respectively.

$\hat{x}_{ap}^{rs}(k), \tilde{x}_{ap}^{rs}(k)$ =the vehicle numbers of complied and unequipped & non-complied travelers on link *a* of path *p* at interval *k,* respectively.

$\hat{V}_{ap}^{rs}(k), \tilde{V}_{ap}^{rs}(k)$ =the cumulative departures of complied and unequipped & non-complied travelers on link *a* of path *p* until interval *k,* respectively.

$\hat{U}_{ap}^{rs}(k), \tilde{U}_{ap}^{rs}(k)$ = the cumulative arrivals of complied and unequipped & non-complied travelers on link *a* of path *p* until interval *k,* respectively.

$t_a(k)$ =the travel time on link *a* for travelers entering this link at interval *k*

The link dynamics equations express the relationship between the flow variables of a link

$$\hat{x}_{ap}^{rs}(k) = \hat{x}_{ap}^{rs}(k-1) + \hat{u}_{ap}^{rs}(k) - \hat{v}_{ap}^{rs}(k) \quad \forall k, \forall rs, \forall a \in p \tag{11}$$

$$\tilde{x}_{ap}^{rs}(k) = \tilde{x}_{ap}^{rs}(k-1) + \tilde{u}_{ap}^{rs}(k) - \tilde{v}_{ap}^{rs}(k) \quad \forall k, \forall rs, \forall a \in p \tag{12}$$

The flow conservation equations for the node

$$\hat{u}_{ap}^{rs}(k) = \begin{cases} \hat{f}_p^{rs}(k) & a \ \ is \ \ first \ \ link \ \ on \ \ path \ \ p \\ \hat{v}_{bp}^{rs}(k) & a \ \ is \ \ after \ \ b \end{cases} \quad \forall k, \forall rs, \forall p \tag{13}$$

$$\tilde{u}_{ap}^{rs}(k)=\begin{cases}\tilde{f}_p^{rs}(k) & a \text{ is first link on path } p\\ \tilde{v}_{bp}^{rs}(k) & a \text{ is after } b\end{cases} \quad \forall k,\forall rs,\forall p \tag{14}$$

Flow propagation constraints are used to described the flow progression over time

$$\hat{V}_{ap}^{rs}(k)=\sum_{j\in[l:0\le j\cdot\boldsymbol{\delta}+T(j)\le(k-1)\cdot\boldsymbol{\delta}]}\hat{u}_{ap}^{rs}(j)\cdot\boldsymbol{\delta} \quad \forall k,\forall rs,\forall p \tag{15}$$

$$\tilde{V}_{ap}^{rs}(k)=\sum_{j\in[l:0\le j\cdot\boldsymbol{\delta}+T(j)\le(k-1)\cdot\boldsymbol{\delta}]}\tilde{u}_{ap}^{rs}(j)\cdot\boldsymbol{\delta} \quad \forall k,\forall rs,\forall p \tag{16}$$

Definitional constraint:

$$x_a(k)=\sum_{rs}\sum_{p}(\hat{x}_{ap}^{rs}(k)+\tilde{x}_{ap}^{rs}(k)) \qquad v_a(k)=\sum_{rs}\sum_{p}(\hat{v}_{ap}^{rs}(k)+\tilde{v}_{ap}^{rs}(k))$$

$$u_a(k)=\sum_{rs}\sum_{p}(\hat{u}_{ap}^{rs}(k)+\tilde{u}_{ap}^{rs}(k)) \quad V_{ap}^{rs}(k)=\sum_{j=1}^{k}(\hat{v}_{ap}^{rs}(j)+\tilde{v}_{ap}^{rs}(j))\cdot\boldsymbol{\delta} \tag{17}$$

$$U_{ap}^{rs}(k)=\sum_{j=1}^{k}(\hat{u}_{ap}^{rs}(j)+\tilde{u}_{ap}^{rs}(j))\cdot\boldsymbol{\delta}$$

Boundary conditions:

$$\hat{x}_{ap}^{rs}(0)=0,\hat{u}_{ap}^{rs}(0)=0,\hat{v}_{ap}^{rs}(0)=0,\tilde{x}_{ap}^{rs}(0)=0,\tilde{u}_{ap}^{rs}(0)=0,\tilde{v}_{ap}^{rs}(0)=0 \tag{18}$$

Now we give the actual path travel time and actual path travel cost functions. The actual travel time to traverse path $p=\{a_1,a_2,\ldots,a_n\}$ for travelers entering into the network during interval k is calculated using the following nested function.

$$t_p^{rs}(k)=t_{a1}(k)+t_{a2}(k+t_{a1}(k))+\cdots+t_{am}(k+t_{a1}+\cdots+t_{am-1}) \tag{19}$$

Here, let $t_{a1}=t_{a1}(k),t_{a2}=t_{a2}(k+t_{a1}(k))$ for short

The schedule delay cost function can be expressed as follows.

$$Sch_s(k)=\begin{cases}\beta\left[t_s-\Delta_s-k-t_p^{rs}(\mathrm{k})\right] & \text{if } t_s-\Delta_s>k+t_p^{rs}(\mathrm{k})\\ \gamma\left[k+t_p^{rs}(\mathrm{k})-t_s-\Delta_s\right] & \text{if } t_s+\Delta_s<k+t_p^{rs}(\mathrm{k})\\ 0 & \text{otherwise}\end{cases} \quad \forall k,m \tag{20}$$

Denote $[t_s-\Delta_s,t_s+\Delta_s]$ as the desired time interval for arrival at the destination s in the network. Where $t_s-\Delta_s$ is the travelers' desired earliest arrival time, $t_s+\Delta_s$ is the desired latest arrival time as the destination s. β,γ is the unit cost of schedule delay early, late at the destination s, respectively.

Therefore, the travel cost of a trip from origin r to destination s on path p for a traveler leaving origin at time interval k is

$$c_p^{rs}(k)=\alpha t_p^{rs}(k)+Sch_s(k) \tag{21}$$

Where α is a convention factor to transform the path travel time into travel cost. In accordance with the empirical results (small, 1982), we assume that $\gamma > \alpha > \beta$ holds.

Up to now, we formulate a multi-class dynamic traffic equilibrium problem as a discrete-time path-based VI model. It has been noted that the aforementioned model is mainly used for evaluating the impacts of ATIS during normal peak hour periods for commuter trips. Therefore, some of the assumptions adopted in the model may be appropriate under certain circumstances such as under recurrent congestion conditions without spillback queue. Additionally, the above multinomial logit model for modeling travelers' simultaneous path and departure time choice behaviors is a very simplistic model that may give unrealistic result of prediction because they neglect the impacts of path overlap. Nevertheless, the explicit analytical expression of the logit model is easy to calculate the ATIS market penetration and compliance rate discussed below. In further studies, a general C-logit, PS-logit and Probit model are used.

2.3 Market penetration and compliance rate model

In this section, we will derive a market penetration model and compliance rate model regarding average travel cost saving. Firstly we assume that travelers' decision-making structure regarding the purchase of ATIS devices and the compliance of ATIS advice is related to average travel cost saving between an OD pair, and can be represented as a tree structure, with purchase choices of ATIS device at the upper level and compliance choices of ATIS advice at the lower level, as shown in Fig.1.

Travelers' purchase choices can be expressed as:

$p=1$ If $U_p>S_p$, $p=0$, if $U_p<S_p$ (22)

Travelers' compliance choices can be expressed as:

$a=1$ If $U_a>S_a$, a=0, if $U_a<S_a$ (23)

The above models assume that a traveler will adopt ATIS advice only when the travelers' purchase utility U_p and compliance utility U_a exceed certain thresholds S_p and S_a. Where $p=1$ is the case that a traveler will purchase an ATIS device, $p=0$ the case that a traveler will not purchase a device, $a=1$ the case that a traveler will comply with ATIS advice and $a=0$ the case a traveler will not comply with ATIS advice.

Assuming that the error term has a Gumbel distribution, traveler purchase utility U_p and compliance utility U_a, threshold S_p and S_a. Can be given by the following equations,respectively

$$U_p = \varpi + \lambda_1 \cdot V_p + \varepsilon_p^u \tag{24}$$

$$S_p = \pi_1 + \varepsilon_p^s \tag{25}$$

$$U_a = \lambda_2 \cdot C_{ave}^{rs} + \varepsilon_a^u \tag{26}$$

$$S_a = \pi_2^{rs} + \varepsilon_a^s \tag{27}$$

$$V_p = \ln(\exp(C_{ave}^{rs}) + \exp(\pi_2^{rs})) \tag{28}$$

Where ϖ is a general parameter capturing other benefits such as convenience of having the device, etc. π_1 is the deterministic terms of the thresholds which show the capital cost to buy and use an ATIS device annually. π_2^{rs} shows the deterministic term of the threshold of the travel cost saving of equipped and complied driver on OD pair rs. λ_1, λ_2 are the value of time, respectively. $\varepsilon_p^u, \varepsilon_p^s, \varepsilon_a^u, \varepsilon_a^s$ are the respective error terms; respectively. Travel cost saving, C_{ave}^{rs}, is defined as the difference of the average travel cost between complied travelers and un-equipped & non-complied travelers on OD pair rs, expressed as:

$$C_{ave}^{rs} = \sum_p \sum_k c_p^{rs}(k) \cdot \tilde{P}_p^{rs}(k) - \sum_p \sum_k c_p^{rs}(k) \cdot \hat{P}_p^{rs}(k) \tag{29}$$

The first term on the right side represents the average travel cost of unequipped & non-complied travelers on each OD pair *rs*, the second term on the right side represents the average travel cost of complied travelers on each OD pair *rs*.

The concurrent probabilities that a traveler will purchase an ATIS device and follow ATIS advice can be calculated in using a nested logit model.

$$P_{pa}^{rs}(p, a | p = 1, a = 1) = \frac{\exp(\varpi + \lambda_1 \cdot V_p)}{\exp(\varpi + \lambda_1 \cdot V_p) + \exp(\pi_1)} \cdot \frac{\exp(\lambda_2 \cdot C_{ave}^{rs})}{\exp(\lambda_2 \cdot C_{ave}^{rs}) + \exp(\pi_2^{rs})} \tag{30}$$

The number of travelers to purchase ATIS device is determined by the following equation on OD pair *rs*.

$$P_p^{rs} = \frac{\exp(\varpi + \lambda_1 \cdot V_p)}{\exp(\varpi + \lambda_1 \cdot V_p) + \exp(\pi_1)} \tag{31}$$

Where P_p^{rs} is the value of market penetration of ATIS for each OD pair *rs*.

The compliance rate of ATIS on each OD pair *rs* can be expressed as follows

$$P_a^{rs} = \frac{\exp(\lambda_2 \cdot C_{ave}^{rs})}{\exp(\lambda_2 \cdot C_{ave}^{rs}) + \exp(\pi_2^{rs})} \tag{32}$$

3. A SOLUTION ALGORITHM

With the ATIS market penetration and compliance rate given by Eqs.(30)and (31),the resulting travel demands of complied travelers and non-complied & unequipped travelers are given below for OD pair *rs*.

$$\tilde{q}^{rs} = q^{rs} \cdot (1 - P_{pa}^{rs}(p, a | p = 1, a = 1)) \tag{33}$$

$$\hat{q}^{rs} = q^{rs} \cdot P_{pa}^{rs}(p, a | p = 1, a = 1) \tag{34}$$

$$P_{pa}^{rs}(p, a | p = 1, a = 1) = P_p^{rs} \cdot P_a^{rs} \tag{35}$$

Here we firstly present a dynamic multi-class equilibrium assignment algorithm (**DMCEAA**) in solving multi-class dynamic stochastic path and departure time equilibrium problems for given $\hat{q}^{rs}$ and $\tilde{q}^{rs}$ $rs \in RS$, then we present a procedure for the determination of the ATIS equilibrium market penetration and compliance rate.

Here, we propose the diagonalisation algorithm to solve multi-class stochastic dynamic simultaneous path/departure time equilibrium problem. The method is similar with these of B.Ran(1996), H.K.Chen(1998) and Han (2003).the method consists of the outer iteration and inner iteration, outer iteration includes the updating estimation of actual link travel time or link inflow rate. Inner iteration calculates the link inflow updating direction and the auxiliary link inflow rate by the method of successive averages. The processes of algorithm are stated as follows.

DMCEAA:

Step 0: Initialization: Set outer iteration counter *m*=1, perform two **stochastic dynamic loading methods** in sequence for the given demand $\hat{q}^{rs}, \tilde{q}^{rs}$ $\forall rs$ according to free flow travel cost, find initial link inflow rate $u_a^m(k), \forall a, \forall k$

Step 1: Inner iteration (MSA)

Step1.0: Initialization: set inner iteration counter *l*=1, $u_a^l(k) = u_a^m(k), \forall a, \forall k$

Step1.1: Calculate link travel cost $c_a^l(k), \forall a, \forall k$ by using $u_a^l(k), \forall a, \forall k$.

Step1.2: Direction finding: perform two **stochastic dynamic loading methods** in sequence for $\hat{q}^{rs}, \tilde{q}^{rs}$ $\forall rs$, according to current actual link travel cost $c_a^l(k), \forall a, \forall k$. This generates auxiliary link flow $u_a^l(k)^*, \forall a, \forall k$.

Step 1.3:Move: find a new flow pattern

$$u_a^{l+1}(k) = u_a^l(k) + \boldsymbol{\lambda}^l (u_a^l(k)^* - u_a^l(k)), \forall a, \forall k$$

Step 1.4: Convergence test of inner iteration: if

$$\sqrt{\sum_a (u_a^{l+1}(k) - u_a^l(k))^2} \Big/ \sum_a (u_a^l(k)) \le \boldsymbol{\gamma}$$

(γ is a predetermined tolerance) or *j* is equal to a given number, then stop; otherwise, go to step 1.1 and set *l*=*l*+1.

Step 2: convergence test of outer iteration: if the convergence criteria are satisfied or *i* is equal to a given number, stop; otherwise, go to step 1 and set *m*=*m*+1.

In this algorithm, stochastic dynamic loading method for the logit-based path and departure time choice is proposed. This network loading method is similar to the method proposed by Dial's STOCH for stochastic static assignment and the method proposed by B.Ran's DYNASTOCH for stochastic dynamic assignment. The method maintains the structure of the DYNASTOCH algorithm, so only deals with reasonable paths, and assigns the demand between OD pair *rs* to the link of the network according to the actual link travel cost. The procedure and proof of the stochastic dynamic loading method see appendix.

The step size λ^l is a predetermined value, we set $\lambda^l = 1/l$, or $\lambda^l = 1$. In order to maintain correct flow propagation constraint, new link flow pattern is calculated by directly updating link choice probability (Han (2003)) or use pure network loading.

3.2 Fixed-point algorithm

An iterative fixed-point algorithm for the determination of the ATIS market penetration and compliance rate is described as below.

Step 0: an initial value of $P_{pa}^{rs(n)}(p,a|p=1,a=1)$ (Written as $P_{pa}^{rs(n)}$ as below) and set n=0.

Step 1: perform **DMCEAA** to get $C_{ave}^{rs}, \forall rs \in RS$

Step 2: update $P_{pa}^{rs(n)}$ by Eq.(30), get new $P_{pa}^{rs(n+1)}$. Meanwhile, calculate the market penetration and compliance rate using Eqs.(31)-(32).

Step 3: convergence criterion.

If $\sum_{rs}\left|P_{pa}^{rs(n+1)} - P_{pa}^{rs(n)}\right| \leq \varepsilon$, then stop. Otherwise go to step 1 and set n=n+1.

4. A NUMERICAL EXAMPLE

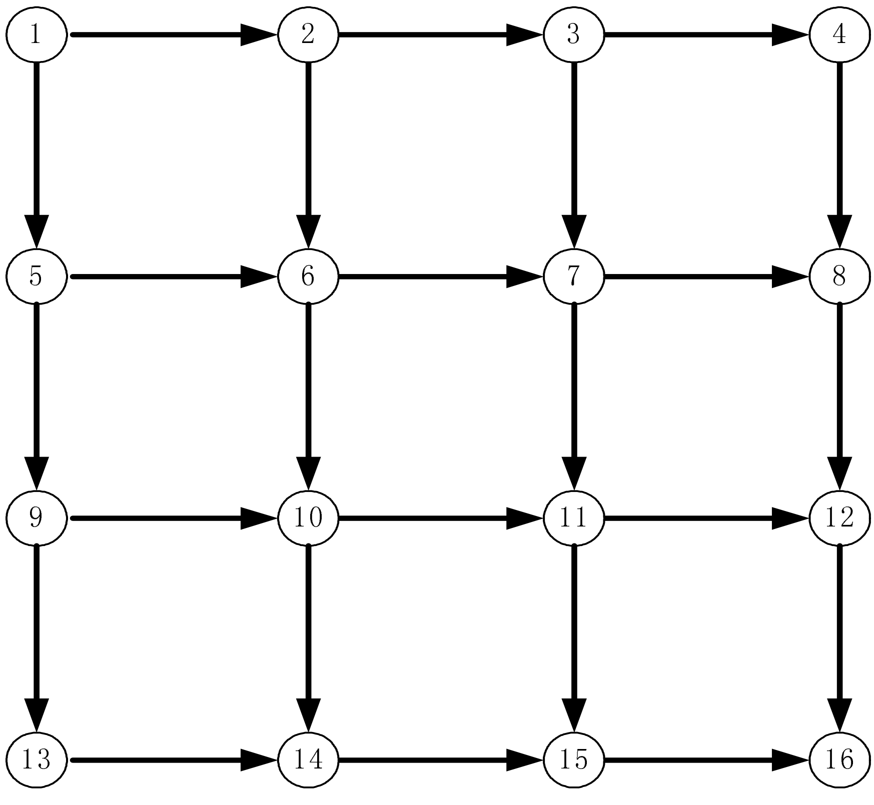

Fig.2 The test network

Table 1. Network input data

Link	α_a	β_a	free flow time	Link	α_a	β_a	free flow time
(1,2)	0.00022	0.0002	0.3	(7,11)	0.0002	0.0001	0.15
(2,3)	0.0003	0.0004	0.2	(8,12)	0.0002	0.0015	0.2
(3,4)	0.0002	0.0011	0.4	(9,10)	0.0002	0.0001	0.15
(1,5)	0.00022	0.0002	0.3	(10,11)	0.0002	0.0001	0.15
(2,6)	0.0005	0.0001	0.25	(11,12)	0.0008	0.0003	0.2
(3,7)	0.0002	0.0001	0.15	(9,13)	0.0002	0.0011	0.4
(4,8)	0.0002	0.0011	0.3	(10,14)	0.0002	0.0001	0.3
(5,6)	0.0005	0.0001	0.25	(11,15)	0.0008	0.0003	0.2
(6,7)	0.0002	0.0001	0.2	(12,16)	0.0002	0.0005	0.15
(7,8)	0.0002	0.0001	0.3	(13,14)	0.0002	0.0011	0.3
(5,9)	0.0003	0.0004	0.2	(14,15)	0.0002	0.0015	0.2
(6,10)	0.0002	0.0001	0.2	(15,16)	0.0002	0.0005	0.15

In this section, **DMCEAA** finds the solution of multi-class stochastic dynamic simultaneous path/departure time equilibrium problem and we perform only one iteration in the inner iteration of the diagonalization method according to Sheffi (1985)'s advice. The market penetration and compliance rate of ATIS is strongly dependent on the travel cost saving generated by the information system. Here we use hypothetical test scenarios to investigate how the travel cost uncertainty and demand level influence the information benefits and the equilibrium market penetration and compliance rate of ATIS.

We choose a simple network for a numerical example. The network scenario is showed in Fig.3, 16 nodes and 24 links. The link travel time function is used $t_a(k) = \chi_a + \alpha_a \cdot x_a(k) + \beta_a \cdot u_a(k)$, Where link free flow travel time χ_a and link parameters α_a, β_a are given in Table 1.There are two OD pairs: (1,16) and (6,16). The feasible path numbers for the two OD pairs are 20 and 6, respectively. We consider a morning peak. We set T be from 6 to 10 a.m. and K=200,so δ=0.02h. This difference in the number of time-dependent paths between the two OD pairs allows for our researches on the spatial impacts of ATIS services. For example, An ATIS between OD pair (1,16) might be effective to help travelers choose the optimal time-dependent path among a large number of alternatives.
The following values of model parameters are used
$\lambda_1 = 0.5\,(\$/\text{h}), \lambda_2 = 0.5\,(\$/\text{h}), \varpi = 0.01\,(\$), \pi_{1(1-16)} = 3.5\,(\$), \pi_{2(1-16)} = 1.75\,(\$), \pi_{2(6-16)} = 1.5\,(\$)$
α=6.4(\$/h) β=3.9(\$/h) λ=15.21(\$/h), Δ=0.25h,k^*=9.0h.

We firstly examine the convergence of the iterative algorithm. Fig. 3 gives the solution of the market penetration against iteration number for the two OD pairs, where parameter $\tilde{\theta}$ is fixed as 0.01, and values of $\hat{\theta}$ for complied travelers and total travel demand q^{rs} vary as shown in the legend. It can be found that the iteration of the algorithm has a fast convergence; Convergence is achieved in about four iterations in all cases. It seems that the demand level and parameter $\hat{\theta}$ have little impacts on the convergence of the algorithm.

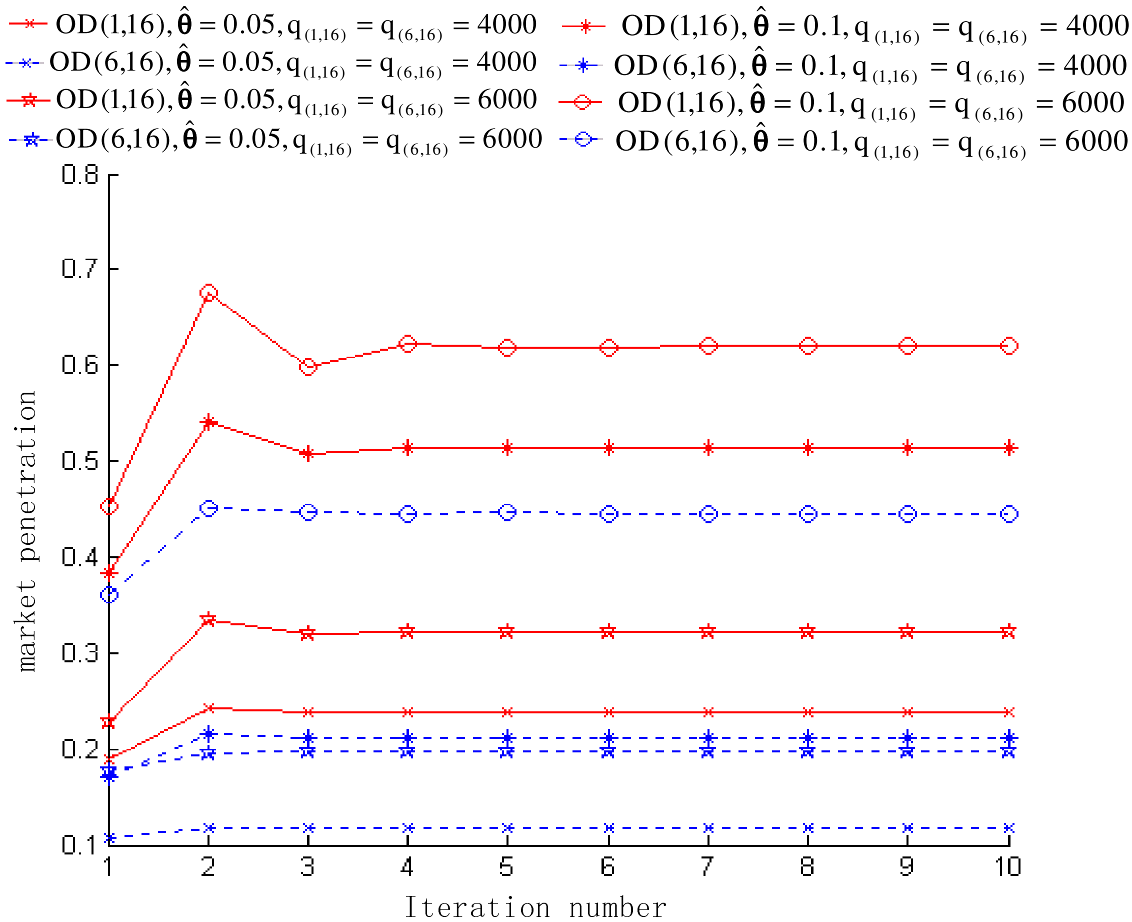

Fig 3. Convergence process of market penetration for different Demand, OD pair and $\hat{\theta}$

Obviously, the market penetration of ATIS is substantially different between the two OD pairs because of their different trip distance and number of available time-dependent paths. Long-distance travelers with ATIS device will generally save more travel cost and thus have more incentive to buy such a device and then follow the advice. Further, It can be seen that the market penetration for OD pair (1,16) is higher than that for OD pair (6,16).

Fig. 4 depicts the convergence process of the compliance rate for the two OD pairs; it can be observed that the convergence properties of the compliance rate are similar to that of the market penetration. Note that the convergence of the algorithm cannot be always guaranteed theoretically. Similar to Yang (1998), Ying (2003), non-convergence has been also observed in our examples where the algorithm may oscillate between two points.

Figures 5 and 6 depict the change of the equilibrium market penetration and compliance rate by varying the values of $\hat{\theta}$ with parameter $\tilde{\theta}$ fixed to be 0.01 for the two OD pairs, respectively. Travel demands for the two OD pairs vary as shown in the legend. As aforementioned,

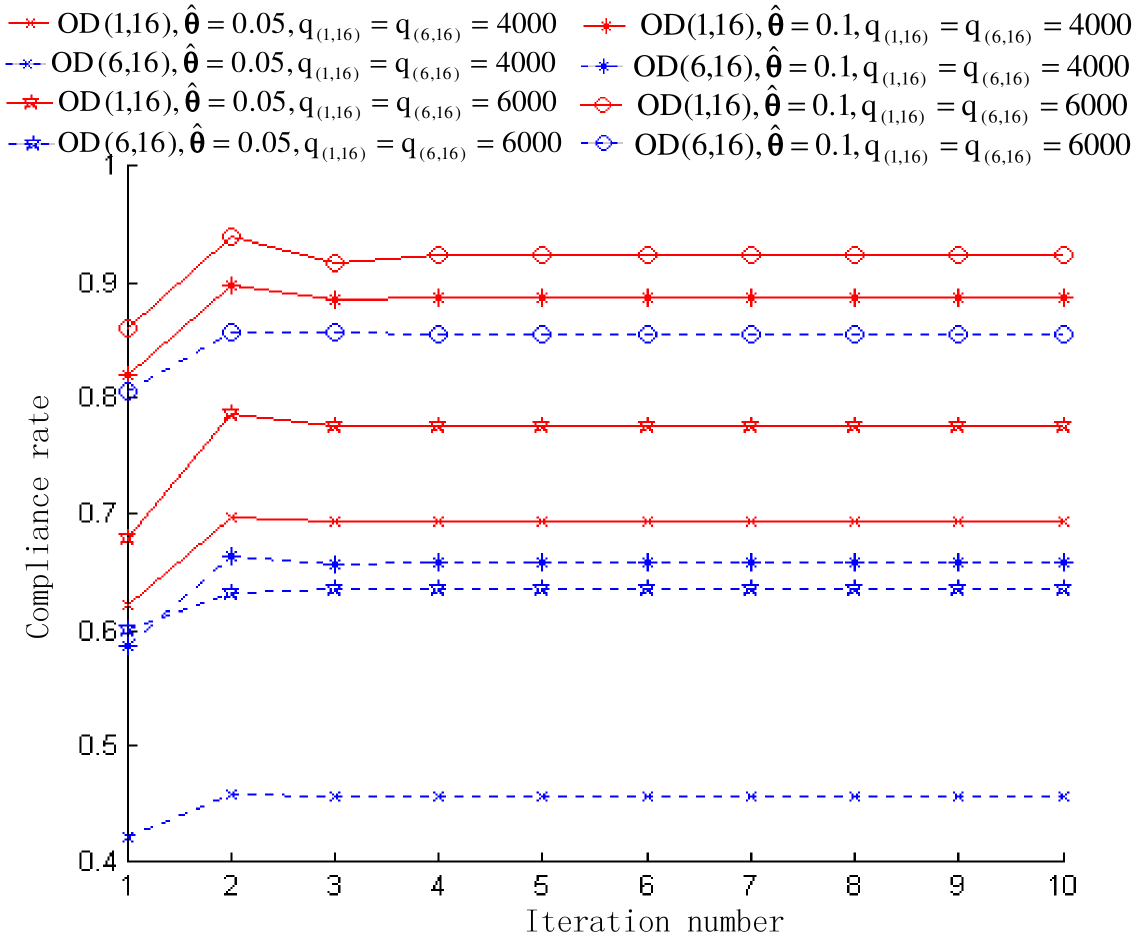

Fig 4. Convergence process of compliance rate for different Demand, OD pair and $\hat{\theta}$

the value of $\hat{\theta}$ shows the degree of complied traveler's uncertainty on travel cost, and also implies the quality of the provided information. It is well known that a larger value of $\hat{\theta}$ implies a higher degree of travelers' certainty on travel cost, and thus travelers have more incentive to buy an ATIS device and comply with ATIS advice. This has been clearly reflected in Fig 5 and 6. Firstly, we can find that travel cost is even not saved at $\hat{\theta} = \tilde{\theta} = 0.01$; there will still be certain market penetration and compliance rate. This models the intrinsic value of purchasing the device and following the advice, including such benefits as convenience, "sense of security." As parameter $\hat{\theta}$ increases, the equilibrium market penetration and compliance rate of ATIS for the two OD pairs increase sharply and gradually approach upper limit. We find the upper limit value of the market penetration and compliance rate for OD pair (1,16) is higher than that for OD pair (6,16). Since the OD pair (1,16) has longer travel distance and more chosen time-based paths compared with the OD pair (6,16), travelers between OD pair (1,16) can obtain more benefits from ATIS. In Figures 5 and 6, it seems that the level of travel

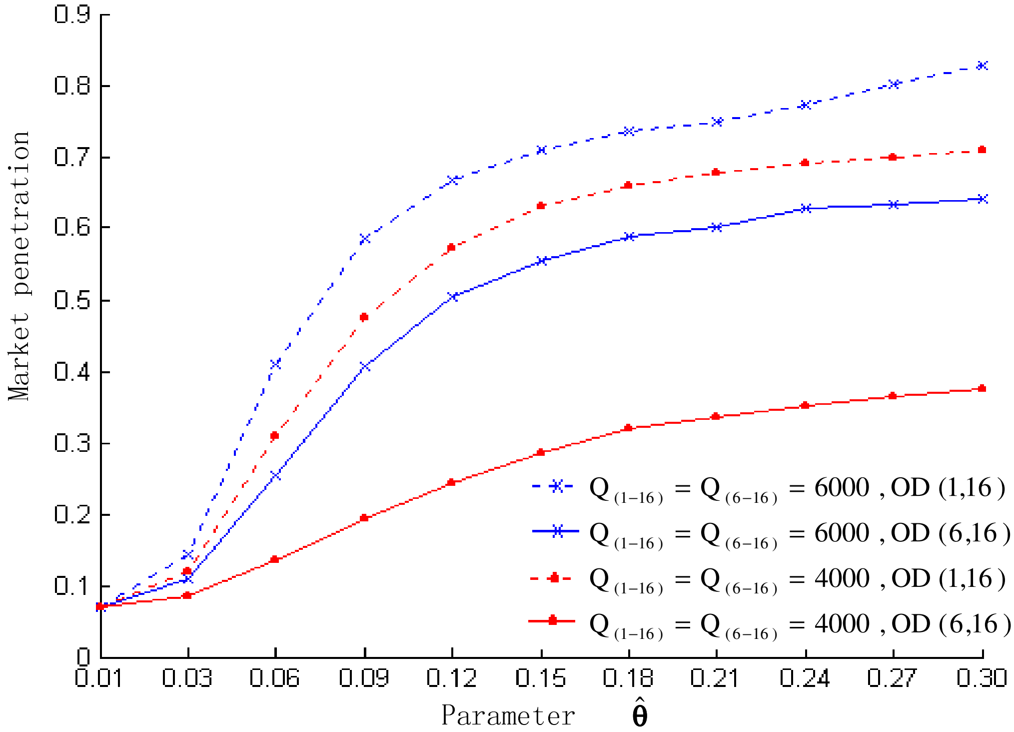

Fig 5. Effect of travel cost uncertainty on market penetration

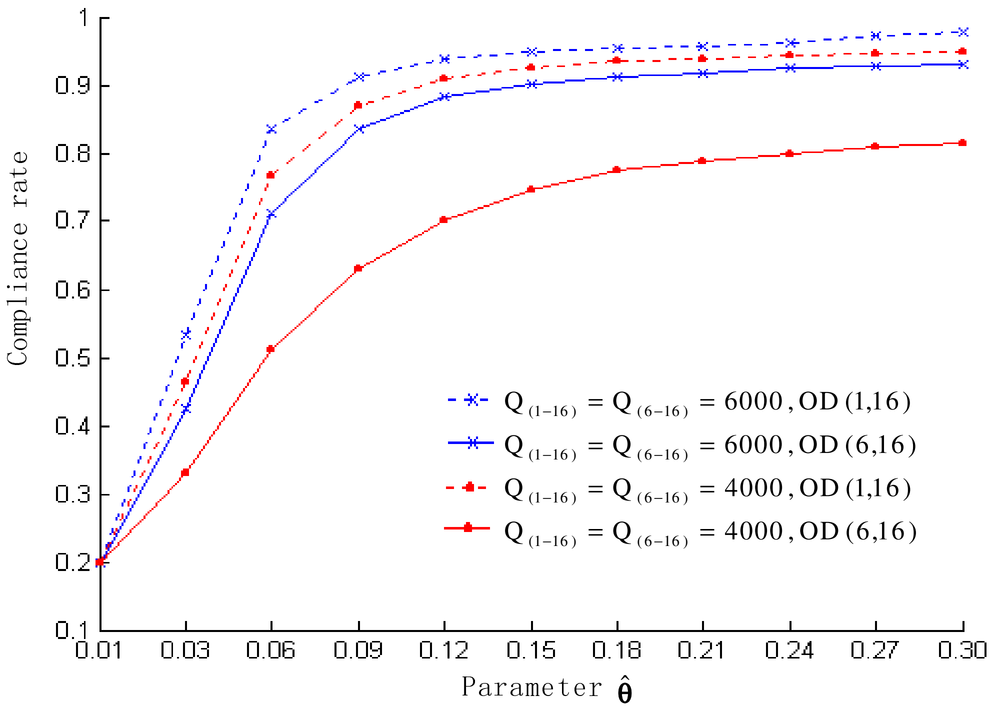

Fig 6. Effect of travel cost uncertainty on compliance rate

demand that governs the congestion level has similar important impacts on market penetration and compliance rate. However, the result is different with that given by Ying (2003) in which that considered the demand level have little impacts on compliance rate of ATIS.

5. Discussions on the parameter calibrated

In the model proposed by this paper, due to our prior limited knowledge, there will inevitably be parameters in the models whose values we do not know. To be able to utilize these models, we will have to rely on empirical observations to enable us to make statistical inferences about the unknown parameters.

There are Many calibrated parameters as listed in this paper, including $\hat{\theta}$,$\tilde{\theta}$ that are relative to the travel choice behavior and ϖ , $\pi_1, \pi_2^{rs}, \lambda_1, \lambda_2$ that are relative to the travelers' compliance and purchase behaviors. Some researchers give some discussions on the parameters calibrated (Hong.Ko 2002a).

Firstly, we propose a procedure to calibrate the parameters $\hat{\theta}$,$\tilde{\theta}$. For a network with actual or measured dynamic flows, we may estimate $\hat{\theta}$,$\tilde{\theta}$ by minimizing the squared errors between the estimated and actual dynamic flows. The minimum function can be expressed as follows: $\min \sum_k \sum_a (x_a(k,\hat{\theta},\tilde{\theta}) - x_a(k))^2$. Where, $x_a(k,\hat{\theta},\tilde{\theta})$ is the estimated link vehicle number at time interval k on link a resulting from multi-class dynamic stochastic path and departure time equilibrium, and $x_a(k)$ is the measured link vehicle number at time interval k on link a.

For the parameters ϖ , $\pi_1, \pi_2^{rs}, \lambda_1, \lambda_2$, we can collect the data by stated preference method based on hypothetical and actual investigations and then use maximum likelihood estimation method in calibrating those parameters(M.Ben-Akiva, 1985). The detail data survey and collection procedures will be important question in future study.

As the implementation scale of ATIS system is enlarged and more experiences accumulated around the world, we can judge whether the above calibration process is proper or not. This will be an important subject for future study.

6. Conclusions

In this paper, we propose a multi-class dynamic stochastic path and departure time equilibrium model with the equilibrium market penetration and compliance rate in an ATIS environment. The nested-logit model is used for modeling the traveler purchase and compliance behaviors. And then we present a stochastic dynamic network-loading algorithm for dynamic simultaneous path/departure time logit choice. The proposed iterative algorithm is applied to a numerical example and demonstrated to have a fast convergence in general, which is similar to static example.

In future studies, the proposed model could be extended in several directions. Firstly, this model assumes that the total demand over the network is fixed, to consider variable demand is one direction. Secondly, the proposed model does not consider or produce system optimal flow patterns, how to combine ATIS with a congestion toll model so as to achieve system optimal pattern would be another important research direction.

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Appendix:

This paper develops a stochastic dynamic network loading method considering the logit-based path and departure time choice. The logit-based path and departure time choice function can be expressed as follows:

$$P_p^{rs}(k) = \frac{\exp(-\boldsymbol{\theta}\cdot c_p^{rs}(k,\cdot))}{\sum_p \sum_k \exp(-\boldsymbol{\theta}\cdot c_p^{rs}(k,\cdot))} \quad \forall rs, p, k \tag{a1}$$

Stochastic dynamic network loading method

In this section, stochastic dynamic network loading method for the logit-based path and departure time choice is proposed. This network loading method is similar to the method proposed by Dial's STOCH for stochastic static assignment and the method proposed by B.Ran's DYNASTOCH for stochastic dynamic assignment. In this study, we consider only the logit model for stochastic dynamic simultaneous path/departure time choice. The method maintains the structure of the DYNASTOCH algorithm, so only deals with reasonable paths, and assigns the demand between OD pair *rs* to the link of the network according to the actual link travel cost.

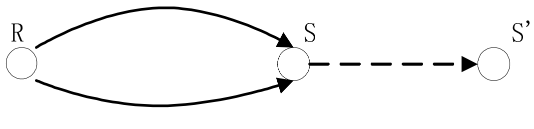

Figure.1 Extended network structure

In order to reflect the effects of schedule delay cost in the algorithm, we extend the original network to include the dummy link with schedule delay cost, $c_m^{s's}(k) = sch_{sm}(k), \forall m$ as shown in Fig.1.

Step 1: Calculation of link likelihood

Compute the minimum actual travel cost $\pi_{is}(k)$ for travelers departing node *i* during time interval *k*. calculate the link likelihood, $L_{(i,j)}(k)$,for each link (*i,j*) during each time interval *k*:

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is} - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{rs} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \in r \tag{a2}$$

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{is} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \notin r \tag{a3}$$

Where equations (a2) and (a3) express the calculation way of the link likelihood when the head node *i* of link (*i,j*) is and isn't the origin *r*, respectively.

$\pi_{is}(k)$ is the minimum travel cost from *i* to *s* by departing the node *i* during interval *k*

π_{rs} is the minimum path travel cost from origin *r* to destination *s* for all departure time. $\pi_{rs} = \min\{c_p^{rs}(k), \forall p, k\} \quad \forall rs$

C_o^{is} is the ideal travel cost from *i* to *s* when there is no flow in the network

$t_{(i,j)}(k)$ is link travel time experienced by travelers entering into link (*i,j*) during interval *k*

$c_{(i,j)}(k)$ is link travel cost experienced by travelers entering into link (*i,j*) during interval *k*.

Step 2: backward pass

By examining all nodes *j* in ascending sequence with respect to $\pi_{is}(k)$ from the destination *s*, calculate $w_{(i,j)}(k)$, the link weight for each link (*i,j*) during each time interval *k*:

$$w_{(i,j)}(k) = \begin{cases} L_{(i,j)}(k) & if \quad j = s \\ L_{(i,j)}(k) \cdot \sum_{(j,k) \in A(j)} w_{(j,k)}(k + t_{(j,k)}(k)) & otherwise \end{cases} \tag{a4}$$

Where *A(j)* is the set of links starting from node *j,* When origin *r* is reached, stop

Step 3: forward pass

Consider all nodes *i* in descending sequence with respect to $\pi_{is}(k)$, starting with the origin *r*. when each node *i* is considered during each time interval *k*, compute the inflow to each link (*i,j*) during each time interval *k* using the following formula:

$$v_{(i,j)}(k) = \begin{cases} q^{rs} \cdot \dfrac{w_{(i,j)}(k)}{\sum\limits_{k} \sum\limits_{(i,k)\in A(i)} w_{(i,k)}(k)} & \text{if} \quad i = r \\ \left\{ \sum\limits_{(k,i)\in B(i)} u_{(k,i)}(k) \right\} \cdot \dfrac{w_{(i,j)}(k)}{\sum\limits_{(i,k)\in A(i)} w_{(i,k)}(k)} & \text{otherwise} \end{cases} \tag{a5}$$

Where, *B(i)* is the set of links ending at node *i.* When destination *s* is reached, stop
The flow generated by the method is equivalent to a logit-based flow independent path/departure time assignment between each OD pair, given the reasonable path set is fixed in order to produce a convergence solution.

Proof of the method
We now prove that the method does generate logit-based flow independent ideal stochastic dynamic simultaneous path and departure time choices between each OD pair. We note that each link likelihood $L_{(i,j)}(k)$ is proportional to the logit probability that link a=(*i,j*) is used during time interval *k* by a traveler chosen at random from among the population of trip-makers between origin *r* and destination *s*, given that the traveler is at node *i* during time interval *k*. The probability that a given path will be used is proportional to the product of all the likelihood of the links comprising this path. Suppose path *p* consists of nodes (*r, 1,2…n, s*) and links *(1,2…h)*. Sub-path p1 includes (*1,2…n, s*) and links (*2…h)*. The probability of traveler choosing path *p* and departure time *k* between origin *r* and destination *s* is $P_p^{rs}(k)$.

$$P_p^{rs}(k) = G \cdot \prod_{a\in p} \left\{ L_{(i,j)}(k) \right\}^{\delta_{ap}^{rs}} \tag{a6}$$

Where *G* is proportionality constant for each OD pair and the product is taken over all links in the networks. Here, $t = k + t_p^{ri}(k)$. The incidence variable δ_{ap}^{rs} ensures that $P_p^{rs}(k)$ include only those links in the *p*th path between origin *r* and destination *s*. Substituting the expression for the likelihood $L_{(i,j)}(k)$ in the above equation, the probability of choosing a particular efficient path-departure time pair becomes

$$\begin{aligned} P_p^{rs}(k) &= G \cdot \exp\left\{\theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k)\right]\right\} \prod_{a\in p_1} \exp\left\{\theta \cdot \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)\right] \cdot \delta_{ap_1}^{rs}\right\} \\ &= G \cdot \exp\left\{\theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k)\right]\right\} \exp\left\{\theta \cdot \sum_{a\in p_1} \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)\right] \cdot \delta_{ap_1}^{rs}\right\} \\ &= G \cdot \exp\left\{\theta \cdot (\pi_{rs} - c_p^{rs}(k))\right\} \end{aligned} \tag{a7}$$

The last equality results from the following summations:

$$\pi_{rs} - \pi_{1s}(k + t_{(r,1)}(k)) + \sum_{a\in p_1}\left[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k))\right]\cdot \delta_{ap}^{rs}$$
$$= \pi_{rs} - \pi_{1s}(k + t_p^{r1}(k)) + \pi_{1s}(k + t_p^{r1}(k)) - \pi_{2s}(k + t_p^{r2}(k)) + \vdots + \pi_{ns}(k + t_p^{rn}(k)) - \pi_{ss}(k + t_p^{rn}(k))$$
$$= \pi_{rs}$$

(a8)

and

$$\sum_{a\in p} c_a(k)\delta_{ap}^{rs} = c_1(k) + c_2(k + t_p^{r1}(k)) + \cdots + c_h(k + t_p^{rn}(k)) = c_p^{rs}(k) \quad \text{(a9)}$$

Since $\sum_p \sum_k P_p^{rs}(k) = 1$

The proportionality constant must equal

$$G = \frac{1}{\sum_p \sum_k \exp\{\theta\cdot[\pi_{rs} - c_p^{rs}(k)]\}} \quad \text{(a10)}$$

Thus

$$P_p^{rs}(k) = \frac{\exp\{\theta\cdot[\pi_{rs} - c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{\theta\cdot[\pi_{rs} - c_p^{rs}(k)]\}} = \frac{\exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta\cdot c_p^{rs}(k)]\}} \quad \text{(a11)}$$

Above equation depicts a stochastic dynamic simultaneous path/departure time choice among the efficient paths connecting OD pair *rs*. The algorithm does generate a stochastic dynamic simultaneous path/departure time choice probability using actual path travel costs.

Now we try to prove the forward pass of the algorithm does generate the results of the logit flow assignment for simultaneous path/departure time choice. Firstly, we transform equation (a11) to the following equation.

$$f_p^{rs}(k) = q^{rs}\frac{\exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta\cdot c_p^{rs}(k)]\}} = q^{rs}\cdot\frac{\sum_p \exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta\cdot c_p^{rs}(k)]\}}\cdot\frac{\exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta\cdot c_p^{rs}(k)]\}}$$
$$= q^{rs}(k)\cdot\frac{\exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta\cdot c_p^{rs}(k)]\}} \quad \text{(a12)}$$

Where

$$q^{rs}(k) = q^{rs}\cdot\frac{\sum_p \exp\{-\theta\cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta\cdot c_p^{rs}(k)]\}} = q^{rs}\cdot\frac{\sum_{(i,k)\in A(i)} w_{(i,k)}(k)}{\sum_k \sum_{(i,k)\in A(i)} w_{(i,k)}(k)} \quad i\in r, \forall rs, k \quad \text{(a13)}$$

The demand between OD pair *rs* during time interval *k* assigns to the network according to the DYNASTOCH algorithm. The equation (a13) is substituted into the forward pass of the DYNASTOCH algorithm; the equation (a1) can be obtained. The main difference (one is $\pi^{rs}(k)$, other is π^{rs}) between the origin link 's link likelihood of the DYNASTOCH algorithm and this algorithm does not influence the calculation results.